A STUDY OF THE INTERRELATION OF POROSITY, PERMEABILITY, AND GRAIN SIZE IN A CONSOLIDATED POROUS MEDIUM

Stuart Allan MacCaffray

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by

Stuart Allen MacCaffrey

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FOREWORD

Despite tremendous technological discoveries and developments, the gas and oil industry of today is still plagued with unresolved and unexplained problems and phenomena, especially in the field of reservoir engineering. One of these is the macroscopic flow of fluids in reservoir rock formations where gas and oil are found and produced. This thesis is a study of but a small facet of the problem and has been conducted with the idea that any work done in this area will help contribute to an increased knowledge of the subject.

The author wishes to acknowledge the sincere and helpful suggestions, advice, and encouragement of Professor Holbrook G. Botset, Head of the Petroleum Engineering Department, University of Pittsburgh, in the projection, progression, and successful completion of this study. Acknowledgment is also made to those authors, investigators, and researchers whose works were utilized for background and basis of the study without which such an undertaking would have been impossible.

I. INTRODUCTION

The Petroleum Industry finds and obtains its raw materials, gas and oil, far beneath the surface of the earth in areas of rock formation called reservoirs. These reservoirs are not, as is usually presumed by the uninformed, large voids filled with gas and oil which, when tapped, may be easily produced. On the contrary, they are formations of porous and permeable rock with small, macroscopic, open spaces between the grains of the matrix material in which the gas and oil is stored; a factor which makes the production of such stored gas and oil difficult and, in some cases, impossible. It is for this reason that the complete study of all conditions and probabilities of fluid flow in porous media becomes an important facet in the production of gas and oil.

The purpose of this study was to investigate and attempt to evaluate the inter-relationships of some of the major physical properties of porous reservoir rock which affect the flow of fluids through it. Although these inter-relationships have been expressed by several different formulations, mentioned later, all designed to elucidate fluid flow, it was thought that a more useful and informative modification could be developed and applied to the problem of correlating permeability, porosity and grain size of porous media to fluid flow through such media. Such a study, designed to reveal the mechanics of macroscopic fluid flow in porous media, should be useful in evaluating some of the many factors which must ultimately be known if a satisfactory solution of the problem is ever to be obtained. Any additional knowledge of these factors should contribute to a more complete understanding of the complexities of gas and oil production from the reservoir.

To the petroleum engineer, especially one interested in reservoir engineering, a very important physical property of reservoir rock is permeability, which can be properly defined as a measure of the fluid-transmitting capacity of a porous material, or as the ability of a fluid to flow within the interconnected pore spaces of a porous material. A second physical property, also equally important, is porosity. In oil and gas reservoirs, porosity represents the percentage of total rock volume which is available for occupancy by either liquids or gasses or both. In connection with the term porosity, care should be taken to distinguish between absolute and effective porosity. Absolute porosity is the percentage of void space with respect to total volume. Effective porosity, on the other hand, is the percentage of interconnected void space in the total volume. A third physical property of reservoir rock is actual grain size and grain size distribution of the matrix material which forms the reservoir rock.

It has been demonstrated by experiment and empirical formulas that permeability, effective porosity, and grain size are interrelated, one depending upon the other to varying degrees. In porous material in which the pores are inter-connected, there exists no definite relation between permeability and effective porosity. These, however, are directly influenced by grain size, in that grain size or particle size and its attendant factors of uniformity of grain size, shape of the grain, manner of grain packing, and the amount of cementing material between the grains (degree of lithification) determine the size and shape of the pore openings, the extent of interconnections of the pore spaces, and the ratio of their volume to the total volume of the rock.

Investigations[1,2,3,6,10] have indicated that actual grain size and effective porosity are excellent criteria for judging permeabilities of various porous media. In much of the literature, variations of permeability and porosity within individual reservoirs have been found to be inter-related to a degree and have been shown by Fettke[1,2] in the case of the Bradford Third sand of the Bradford field of Pennsylvania to have these relationships

$$K = 6.49 \times 10^{-10} (\phi)^{5.65}$$

and

$$K = 5.1 \times 10^{-10} (\phi)^{6.9}$$

where K is the permeability and ϕ is the porosity. The first formula was determined from earlier experiments and the second from later experiments. However, these relationships are average and do not apply in some cases or throughout the complete possible range of permeability and porosity. Here again grain size, with its attendant factors, directly affects these relationships.

In making determinations of the values of permeability, effective porosity and grain size diameter, the first two are now very easily and effectively carried out by means of simplified, standardized methods which give fairly accurate results. In the case of average grain size diameters, this determination becomes complicated by the particle itself, in that the particle is usually irregular in size and shape, and no known methods are available for defining such a particle in geometric terms. This problem becomes more complicated and formidable in the case of an aggregate of irregular particles, but has been successfully solved by the use of statistical methods for evaluating surface area and volume of the aggregates.

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[1]References are listed in the Bibliography.

Such methods are based on the measurement of the so-called average grain diameter by relating the irregular particle to an equivalent geometric figure such as a circle or a sphere from which a diameter can be calculated. These methods determine surface area and volume from the number and weight of the particles, using as a basis the theory of dimensions which requires that the surface area be proportional to the square, and the volume be proportional to the cube of the dimensions used.

For the determination of particle size to have physical significance, that is, to relate particle size to definite physical properties such as surface or volume, the arithmetic or geometric mean and the median diameter determinations are superseded by statistical diameters. Several formulas have been developed and used[15] to designate these statistical diameters in terms of various physical properties such as mean diameter (length), mean volume diameter, mean surface diameter, mean volume-surface diameter, and weight mean diameter. The average grain diameters used in this study are mean volume diameters, based on the average volume and defined to be that diameter whose volume divided into the total volume would give the number of particles. If d is the average particle diameter and V is the total volume, a relationship exists as

$$V = \sum nd^3$$

where n is the number of particles of a given size. If d_v^3 represents the volume of an average particle, and $\sum n$ is the total number of particles then

$$n = \sum nd^3/d_v^3$$

and

$$d_v = (\sum nd^3/\sum n)^{1/3}$$

II. HISTORICAL BACKGROUND

Investigation of the flow of fluids through porous material at low pressures was initiated by Darcy[3] in 1856. These studies consisted of a series of experiments on the flow of water through filter beds from which Darcy obtained an empirical formula showing the rate of flow to be very nearly proportional to the pressure drop per unit length of porous medium. In its elementary form, this formula,

$$v = -(K/\mu)(\Delta P/\Delta L)$$

indicates that the fluid permeability, K, of a porous material proportionately relates the velocity, v, of flow of a fluid of μ viscosity to the pressure differential, ΔP, causing this flow over a length of ΔL. Since ΔP is measured between the outflow and inflow ends, it is negative as indicated in the formula above.

The proportionality constant K is considered to be a specific property of the porous material, empirically independent of the dimensions of the material, the pressure differential exerted on the fluid flowing, and the viscosity of the fluid. For this reason, K may be expressed in terms of other measurable physical properties of the porous material, such as porosity and grain size. Since the formulation of Darcy's equation and from it, his law, many investigators and researchers have attempted to show relationships of porosity, permeability, and grain size.

In 1880, Seelheim[17] first introduced grain size into the relationship by showing that the rate of flow was proportional to the square of the average grain diameter. This would indicate a decrease in permeability as sand grains decrease in size.

The next investigator to utilize grain size in flow relationship formulas was Hazen[4] who, in 1892 and 1893, proposed the formula based on studies of sand filters,

$$q = K_H (d_H)^2 (h/L)$$

where K_H is a permeability constant, h/L is the pressure drop through the filter per unit-depth, and d_H is the "effective" diameter of sand particles having uniformity coefficients of less than 5. "Effective" grain size is defined as the opening which will just pass ten per cent of the particles (by weight) and uniformity coefficient is defined as the ratio of the size opening which will pass 60 per cent of a sample being screened, to the size which will just pass 10 per cent. When the uniformity coefficient is low, as 5 or less mentioned above, the particles are more or less uniform in size, but as the uniformity coefficient increases to higher values, the particle sizes become widely distributed.

In using Hazen's equation, if q is expressed in cubic feet of water per day per square foot of filter area, d_H in millimeters, the loss of head, h, in feet of water, and L in feet, then the values of K_H range from 1300 to above 4000 depending on the cleanliness of the sand, with the usual limits of K_H for ordinary sands ranging between 2300 and 3300. Hazen's equation has been widely accepted and utilized by sanitary engineers and is of considerable historical interest since it represents the first attempt to recognize the importance of particle size and a method for its satisfactory representation. Wavis and Wilsey[5] retested Hazen's formula with the following results:

$$q = 2300\, d_H^2 \left(\frac{T + 10}{60}\right)(h/L)$$

and

$$q = 980 (d_{34})^2 (\phi/40)^6 \left(\frac{T + 10}{60}\right)(h/L)$$

where T is the temperature of the water in degrees Fahrenheit, d_{34} is the

screen size in millimeters through which just 34 per cent of the particles (by weight) pass, and ϕ is the porosity introduced into the second formula.

Slichter,[6] in 1899, published the accounts of his theoretical analysis of fluid flow through an ideal homogeneous packing composed of spheres of uniform size in which he was the first to introduce the effect of packing as a factor influencing permeability, which gave rise to expressing the average pore area in terms of the diameter of the sphere. Slichter derived a modified Poiseuille relationship in his equation

$$q = 10.2Ad^2h/K_s\mu L$$

where q is expressed in cc of water per second, A is the cross-sectional area of the packing in square centimeters, h is the pressure differential in centimeters of water, d is the diameter of the spheres in centimeters, K_s is a packing constant dependent on porosity, μ is the viscosity of the fluid flowing in poises, and L is the thickness along the direction of flow in centimeters. In terms of Darcy's permeability, Slichter's formula would read

$$K = 10.2d^2/K_s$$

For various packings, tabulations have been made of the different values of the packing constant K_s shown as a function of porosity for porous material packed with spheres of equal diameter.

Zavis and Wilsey[5] developed a formula

$$1/K_s = 0.05(\phi/40)^{3.3}$$

for use in the determination of the values of K_s where ϕ is the porosity. King[7] recognized the importance of Slichter's formula by using it to calculate the average diameter of irregular particles. It is important to note here that the average diameter of irregular particles determined in this manner is the diameter of the particles such that if all of them were

water than in milliliters through which just [illegible] per second of the [illegible] (by volume) pass, and d is the porosity introduced into the second formula.

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$$q = 10.2 h d^2 A / k_s \mu L$$

where q is expressed in cc of water per second, A is the cross-sectional area of the packing in square centimeters, h is the pressure differential in centimeters of water, d is the diameter of the spheres in centimeters, k_s is a packing constant dependent on porosity, μ is the viscosity of the fluid (fluidity) in poises, and L is the thickness along the direction of flow in centimeters. In terms of Darcy's permeability, Slichter's formula would read

$$K = 10.2 d^2 / k_s$$

For various packings, calculations have been made of the different values of the packing constant k_s given as a function of porosity for [illegible] theoretical packed with spheres of equal diameter.

[illegible] and Wilson[7] developed a formula

$$1/k_s = [illegible]^{3.3}$$

for use in the determination of the values of k_s where f is the porosity. [illegible][7] recognized the importance of Slichter's formula by using it to calculate the average diameter of irregular particles. It is important to note here that the average diameter of irregular particles determined in this manner is the diameter of the particles such that if all of them were

of this diameter, the packing would have the same permeability as it actually has at a given temperature and porosity.

For the ultimate in permeability, porosity, and grain size relationships, it remained for Kozeny,[8] in 1927, to develop and publish his important empirical formula

$$v = (C\phi^3 g \Delta P/uS^2 L)(L/L_e)$$

where C is a form factor dependent on the shape of the pores, ϕ is porosity, g is the acceleration constant, ΔP is the pressure differential in grams per square centimeter, u is the viscosity of the fluid flowing in poises, S is the specific surface area in square centimeters per cubic centimeter, L is the thickness of the porous medium in centimeters, L_e is the longer path of flow in centimeters through the thickness L, and L/L_e is a reduction factor for the relationship $\Delta P/L$. Since $S = S_o(1 - \phi)$ the above equation can be rewritten in terms of S_o, which is the specific surface area per unit volume of matrix material, cm^{-1}, as

$$v = (C\phi^3 g \Delta P/uS_o^2(1 - \phi)^2 L)(L/L_e)$$

Unaware that Kozeny had developed the aforementioned relationship, Fair and Hatch,[9] in 1933, developed an almost identical formula by following a line of reasoning similar to that pursued by Kozeny. This equation is

$$v = g\phi^3 \Delta P/5u(1 - \phi)^2 LS_o^2$$

It can be seen that the factors $(C \cdot L/L_e)^{-1}$ in the Kozeny equation have been replaced by the constant 5, believed by Fair and Hatch to apply to normal, rather compact, unconsolidated porous material.

Carman[10,11] in 1937, presented an improved derivation of the Kozeny and Fair and Hatch equations. Using as a basis the general equation for flow through pipes

$$v_e = gm^2 \Delta P/k_o u L_e$$

where v_e is the average velocity, g is the acceleration constant, m is the hydraulic radius, defined as the volume of fluid in the pipe per surface area in contact with the fluid, ΔP is the pressure differential, k_o is a shape factor and is the reciprocal of Kozeny's form factor C, u is the viscosity of the fluid flowing, and L_e is the length of flow path, Carman set about to substitute or modify the equation factors so that the equation would represent flow of fluid through porous media. For v he substituted v/ϕ, later to modify it by the relationship L_e/L for flow over the flow path, L_e, greater than the L dimension of the medium. Then he substituted for m the relationship of $\phi/S_o(1 - \phi)$, and applying the reducing factor L/L_e to the expression $\Delta P/L$, he arrived at the following rearrangement:

$$v = (g\phi^3 \Delta P/uS_o^2(1 - \phi)^2 L)(L/L_e)^2$$

In a series of experiments using a very wide range of particle shapes in unconsolidated media, Carman found that the parameter $(C \cdot L/L_e)^{-1}$ of Kozeny's equation and the parameter $k_o(L_e/L)^2$ of his equation both averaged about 5. In the modified Kozeny-Carman equation these parameters are expressed as k, with an assigned value of 5, shown by

$$v = g\phi^3 \Delta P/uS_o^2 k(1 - \phi)^2 L$$

Wyllie,[12] in his paper on the historical development of the Kozeny-Carman equation, discusses the more recent determinations by investigators of values greater than 5 for the Kozeny-Carman constant k when the formula has been used in connection with measurements of flow through consolidated media. Since the k_o shape factor of the Kozeny-Carman equation has been held by most investigators to be constant, the higher values of k have been attributed to the $(L_e/L)^2$ factor which Rose and

Bruce[13] have called the tortuosity factor, T. Several determinations were made by Rose and Bruce for the value of k for various consolidated media of low permeability where values were obtained as high as 100. Others concur with Rose and Bruce, all indicating that the constant for consolidated media varies to higher values and that these higher values are attributable to the tortuosity function.

There has been some disagreement with the concept of increasing values of the Kozeny-Carman constant in connection with consolidated media. H. E. Rose[14] believes that errors arising from the use of 5 as a value for the constant will be far less than those arising from uncertainties in other variables, especially the porosity function, expressed as $\phi^3/(1 - \phi)^2$ in the Kozeny-Carman equation. He has shown, by plotting relative resistance versus per cent porosity, that at porosities lower than 40 per cent there are increasingly different values of relative resistance to flow for various equations which all contain a porosity function of some sort. DallaValle[15] has shown a similar plot of relative permeability versus per cent porosity for several different investigators' formulas, all containing a porosity function. This also indicated that at porosities lower than 40 per cent the porosity function does not remain constant. It has not been possible to test the porosity function $\phi^3/(1 - \phi)^2$ in consolidated media because it is not readily possible to vary porosity and keep the specific surface area constant.

A recent modification has been made in the Kozeny equation by Wyllie and Spangler[16] who have combined the Kozeny equation with properties of the capillary pressure desaturation curve to obtain the formula

$$K = (\gamma^2/2.5F^2\phi) \int_0^1 dS_w/P_c^2$$

where K is the permeability coefficient, γ is the interfacial tension, F

is the formation factor, ϕ is the porosity fraction, S_w is the fractional wetting phase saturation, and P_c is the capillary pressure. This equation introduces new physical factors which are more easily measurable in laboratory determinations and consequently should further the continuing search for more accurate and useful relationship formulations for fluid flow through porous media.

III. PROCEDURE

For the purpose of carrying out this study, data of experimental and calculated values for permeability, together with accompanying values for porosity and average grain size, were selected from Fettke's[1,2] studies of the Bradford Oil Field, Bradford, Pennsylvania. These data had been derived from test determinations made on representative core samples obtained from various sections of the field at random depths in the Bradford Third sand. The Bradford Third sand is an excellent representative of a consolidated porous medium whose general characteristics are low permeability and porosity and quite fine grain size with irregular shape and size distribution and a moderate to high amount of lithification. Since practically all oil and gas reservoirs are located in consolidated rock formations, a representative sample of this type of porous medium was chosen for application to this study.

As a first approach to the problem, an investigation was made of permeability, porosity and grain size relationships when applied to an equation modified by the author from one developed by Traxler and Baum[18] from the Poiseuille equation of flow through pipes. This equation was expressed by Traxler and Baum as:

$$D_e = (32q\mu L/A\phi\Delta P)^{\frac{1}{2}} \qquad (1)$$

where D_e is the effective pore diameter in centimeters, q is the rate of flow in cubic centimeters per second, μ is the viscosity of the fluid flowing in poises, L is the length of flow path in centimeters, A is the cross-sectional area in square centimeters, ϕ is the porosity, and ΔP is the pressure differential in dynes per square centimeter. It can be readily seen that the function $q\mu L/A\Delta P$ of this equation will be equivalent

to the permeability coefficient K, in darcies, when the units are changed to those of the Darcy equation. Thus, $K = (q100\mu L/A9.8692 \times 10^{-7}\Delta P)$ or $9.8692 \times 10^{-9} K = q\mu L/A\Delta P$. Substituting in equation (1) above gives:

$$D_e = 5.619 \times 10^{-4}(K/\emptyset)^{\frac{1}{2}} \qquad (2)$$

where K is the permeability coefficient in darcies.

Table I, Appendix I, has been presented to show calculations which resulted from the application of the permeability and porosity data selected for this study to equation (2) above, solving for D_e, the effective average pore diameter. Nutting[19] has stated that the effective average pore diameter is roughly one-fifth of the average grain diameter. Figure 1, Appendix II, shows a plot made from the data in Table I of d, the average grain diameter, versus D_e, the effective average pore diameter, resulting in a straight line whose slope, which represents the ratio of d/D_e, is approximately 25. The value of this ratio is considerably greater than the one of 5, $(1/5d = D_e)$, proposed by Nutting. It should be explained here that only the results of the calculations using Fettke's 1934 data were used in plotting Figure 1. The results of the calculations using Fettke's 1938 data produced no conclusive plot.

As a second approach to the problem, an investigation was made of permeability, porosity and grain size relationships when applied to a formulation derived by equating the Kozeny-Carman and the Darcy equations. This second study was made on the basis of the specific surface area, S_o, of porous medium per unit volume of matrix material. By taking the basic Kozeny-Carman equation solved for S_o^2 one obtains:

$$S_o^2 = \frac{g\emptyset^3 A\Delta P}{5(1-\emptyset)^2 q\mu L} \qquad (3)$$

It can be seen that in equation (3) on the preceding page the function $(A\Delta P/q\mu L)^{-1}$ is equivalent to the K of Darcy's equation when the units of μ and ΔP are changed from poises to centipoises and from grams per square centimeter to atmospheres respectively. Thus, $K = q100\mu L/(A9.678 \times 10^{-4}\Delta P)$ or $9.678 \times 10^{-6} K = (A\Delta P/q\mu L)^{-1}$. Substituting in equation (3) gives:

$$S_o^{\ 2} = \frac{g\,\phi^3}{5(1-\phi)^2 9.678 \times 10^{-6}K} \qquad (4)$$

Since the acceleration constant, g, is 980 centimeters per second squared, equation (4) above can now be resolved as

$$S_o = 4501 \cdot \frac{\phi}{1-\phi} \cdot (\phi/K)^{\frac{1}{2}} \qquad (5)$$

which is the equation used in this second approach to the problem under study.

Table II, Appendix I, has been prepared to show calculations resulting from the application to equation (5) above of the permeability and porosity data selected for this study. It has been shown by Herdan[20] that $S_o = Z_s/Z_v d$ for irregular particles where Z_s/Z_v is the ratio of the surface to volume factors and d is the average grain diameter. In this ratio, $Z_s = S/d_s^{\ 2}$, where S is the surface area and d_s is the statistical particle diameter based on surface area, and $Z_v = V/d_v^{\ 3}$, where V is the volume and d_v is the statistical particle diameter based on volume. The ratio Z_s/Z_v has been indicated by Fair and Hatch[9] to be a useful measure of particle shape and has been given a value of 6.0 for spheres, 6.1 for rounded particles, 6.4 for worn particles, 7.0 for sharp particles, and 7.7 for angular particles. Figure 2, Appendix II, shows a plot made of

the values in Table II of S_o versus $1/d$ which roughly form straight lines with a slope of 143 for line 1 using Fettke's 1934 data and a slope of 59 for line 2 using Fettke's 1938 data. The slope normally should represent the values of the ratio Z_s/Z_v which lie somewhere between 6.0 and 7.7.

As a third approach to the problem, an investigation was made of permeability, porosity and grain size relationships when applied to a formulation derived by equating the Kozeny-Carman and the Darcy equations, as previously mentioned, except that the formulation was modified for the use of average grain size in lieu of specific surface area. Since S_o is proportional to $1/d$, S_o^2 can be replaced by $1/d^2$ in equation (4) above. There is also a new constant, k', introduced into the equation which is equivalent to the function $1/(Z_s/Z_v)^2 5$ where 5 is the Kozeny-Carman constant and Z_s/Z_v is the ratio of the surface to volume factors. The resulting equation is expressed as:

$$\frac{1}{d^2} = \frac{k' \, g \, \phi^3}{(1 - \phi)^2 9.678 \times 10^{-6} K} \qquad (6)$$

By substituting 980 centimeters per second squared for g in equation (6) and rearranging to solve for k', the equation can now be expressed as:

$$k' = \frac{K \, (1 - \phi)^2}{d^2 \, \phi^3 \, 1.013 \times 10^8} \qquad (7)$$

Table III, Appendix I, has been prepared from calculations derived by applying the data used in this study to the factors of equation (7); namely, K/d^2 and $1.013 \times 10^8 \phi^3/(1 - \phi)^2$. The values of these factors were plotted as K/d^2 versus $1.013 \times 10^8 \phi^3/(1 - \phi)^2$, shown by Figure 3, Appendix II, to be a straight line with a slope of 0.0001052 in the case of line 1 using Fettke's 1934 data and a curve with a variable slope of

unknown value in the case of line 2 using Fettke's 1938 data. Line 2 could not be expressed as a straight line by replotting on semi-log or log-log graph paper. The slope of line 1 of Figure 3 is equal to the constant k' of equation (7) on the preceding page.

As previously stated, k' is equivalent to the function $1/(Z_s/Z_v)^2 5$. Using the value of the slope determined for line 1 of Figure 3 for k', the value of Z_s/Z_v as obtained from the relation $Z_s/Z_v = (1/k'5)^{\frac{1}{2}}$ is 43.5. Normally, values for Z_s/Z_v lie between 6.0 and 7.7 as previously mentioned.

In the two preceding investigations, the porosity function, $\phi^3/(1 - \phi)^2$, although it varies with the value of porosity, is considered a constant factor in that it retains the same fixed relationship regardless of the source of data. For a fourth approach to the problem, an investigation was made of the permeability, porosity and grain size relationships when applied to a formulation developed to express the porosity function as a variable, dependent upon the source of data, with the permeability and grain size being constant factors regardless of the source of data. Thus, the porosity function would be a constant factor only for a particular reservoir formation or even a particular section of the reservoir formation.

To develop this formulation, the basic Kozeny-Carman equation was recast to include particle diameter, giving:

$$q = \frac{g\ k'\ A\ \phi^3\ d^2\ \Delta P}{\mu\ L\ (1 - \phi)^2} \qquad (8)$$

whose units have been previously defined. Solving for the porosity function gives:

$$\frac{\phi^3}{(1-\phi)^2} = \frac{q \mu L}{A \Delta P\, g\, k'\, d^2} \tag{9}$$

Since the function $q\mu L/A\Delta P$, when expressed in terms of the Darcy equation, is equal to the permeability coefficient K of Darcy's equation, substitution of $9.678 \times 10^{-6}K$ for $q\mu L/A\Delta P$ can be made in equation (9) above giving

$$\frac{\phi^3}{(1-\phi)^2} = \frac{9.678 \times 10^{-6}K}{g\, k'\, d^2} \tag{10}$$

Assuming that the constant k' would be reflected in a recasting of the porosity function, a new expression of this function can be substituted for $k'\phi^3/(1-\phi)^2$ in equation (10) above giving

$$(\phi)^n = K / 1.013 \times 10^8 d^2 \tag{11}$$

or, by rearrangement,

$$K = 1.013 \times 10^8\, d^2\, (\phi)^n \tag{12}$$

where n is a constant applicable only to a particular formation or section of a formation, or a porous medium.

Table IV, Appendix I, has been prepared from calculations derived from the application of Fettke's 1934 data to the functions of equation (12) above. Figure 4, Appendix II, is a log-log plot of these functions, $K/d^2/1.013 \times 10^8$ versus ϕ, which forms a straight line with a slope of 8. Since

$$n = \frac{\log K/d^2/1.013 \times 10^8}{\log \phi}$$

then n = 8, the slope of the straight line.

IV. COMMENT AND DISCUSSION OF RESULTS

Before discussing the results, it should be pointed out here that the values of porosity data used in this study represent absolute porosity. However, Fettke[2] has shown that in a majority of cases the difference between absolute porosity and a lower value for effective porosity in the Bradford Third sand is but a matter of 0.1 per cent average. There are a few isolated cases of a deviation slightly greater than 0.1 per cent up to about 1.0 per cent. For purposes of this study, the values of effective porosity were assumed to be the same as those for absolute porosity. This fact can explain some of the slight deviations of the points used to plot the various lines and curves shown by Figures 1, 2, 3 and 4. Other deviations of the points can best be explained by the fact that at the time the data were generated, the accuracy of the determinations of values for permeability, porosity and grain size was not nearly as good as that of the present day, due to greatly improved and standardized methods for determining such values.

The results of the author's first investigation clearly show that the average pore diameter in a rather fine-grained, consolidated medium decreases considerably from the average for unconsolidated media, stated by Nutting[19] to be roughly one-fifth of the average grain diameter. This can be best explained by the non-uniformity of particle size and shape, the fine grain size, and by the degree of lithification or cementing found in consolidated media. It is considered by the author that the latter exerts the greatest effect upon average pore diameter since cementing to any extent tends to restrict, reduce and plug the pore channels of the rock. To utilize this method of correlation of permeability, porosity

and grain size, it would be necessary to determine the relationship between average effective pore diameter and average grain diameter for each particular reservoir or section of the reservoir.

In the second approach to the problem, the results of the investigation indicate that the value of the Kozeny-Carman constant for consolidated media is much higher than the assigned value of 5. By way of example, presume the consolidated medium to be composed generally of sharp particles where the value of the ratio of surface to volume factors, Z_s/Z_v, is 7.0. It has been previously shown that $S_o = Z_s/Z_v d$. Using as a selection from Fettke's 1934 data, d = .0093 centimeter, k = .0034 Darcy, and ϕ = .145, it is possible, by resolution of equation (4) of section III of this study, to solve for a value of the Kozeny-Carman constant in lieu of the value of 5 normally used. S_o is resolved as 7/.0093 or 753. Solving for the unknown Kozeny-Carman constant using the above values in the equation gives a value of approximately 220. By using some of the other sets of values for permeability, porosity, and grain size from the 1934 data to solve for the Kozeny-Carman constant in the same manner as described above gives values for the constant in the general vicinity of 220, with some variations. The results of similar application of Fettke's 1938 data were not as conclusive, with the values showing a much wider variation, 30 to 107, for the Kozeny-Carman constant.

As in the second approach, the third approach to the problem also indicated that the Kozeny-Carman constant for consolidated media is much higher than 5. It has been previously stated in Section III of this study that the slope of line 1 of Figure 3, Appendix II, was 0.0001052 and equivalent to the function $1/(Z_s/Z_v)^2k$ where k is the Kozeny-Carman constant. If, for example, it is again assumed that Z_s/Z_v has a value of

and grain size, it would be necessary to determine the statistically median average effective pore diameter and average grain diameter for each particular reservoir or section of the reservoir.

In the second approach to the problem, the results of the investigation indicate that the value of the Kozeny-Carman constant for consolidated media is much higher than the accepted value of 5. By way of example, presume the consolidated medium to be composed generally of sandy particles where the value of the ratio of surface to volume factors, f_s/f_v, is 7.0. It has been previously shown that $D_p = (f_s/f_v)d$. Using as a calculation from Fatt's 1956 data, $d = .0093$ centimeters, $k = .0034$ darcy, and $\phi = .115$, it is possible, by rearranging of equation (4) of Section III of this study to solve for a value of the Kozeny-Carman constant in lieu of the value of 5 normally used. D_p is obtained as 7/.0093 or 753. Solving for the unknown Kozeny-Carman constant using the above values in the equation gives a value of approximately 120. By using some of the other sets of values for permeability, porosity, and grain size from the 1956 data to solve for the Kozeny-Carman constant in the same manner as described above gives values for the constant in the general vicinity of 120, with some variations. The results of similar application of Botset's 1950 data were not as conclusive, with the values showing a much wider variation, 70 to 107, for the Kozeny-Carman constant.

As in the second approach, the third approach to the problem also indicated that the Kozeny-Carman constant for consolidated media is much higher than 5. It has been previously showed in Section III of this study that the slope of line 1 of Figure 5, Appendix II, was 0.0001252 and equivalent to the function $1/(f_s/f_v)^2 K$ where K is the Kozeny-Carman constant. If, for example, it is again assumed that f_s/f_v has a value of

7.0 for this medium, it is possible to solve for a value of other than 5 for the Kozeny-Carman constant. In this case, the value was resolved as 194 for the constant. Since a small deviation in the drawing of line 1 of either Figure 2 or 3 results in a change in the value of the slope of the line, it can be presumed that this fact caused the difference in values for the Kozeny-Carman constant determined by the two investigations. Therefore, it can also be presumed that in the case of the 1934 data used the average value for the Kozeny-Carman constant lies somewhere between 194 and 220. No similar success was achieved in using Fettke's 1938 data. It is believed that the 1938 data were not truly representative of the reservoir sand nor were they in themselves complete enough to permit drawing any conclusions.

The fourth approach to the problem was an attempt to show the porosity function as a variable rather than a constant factor in lieu of a variable for the Kozeny-Carman constant as shown by the second and third investigations. An application of the 1934 data to equation (12) of part III indicates that the n of the porosity function, ϕ^n, is approximately equal to 8, as determined from the slope of the line plotted in Figure 4, Appendix II, with higher and lower variations in some cases. Since the porosity function is an arbitrary selection by the author, it follows that this particular function, being an exponential, could have a variety of forms all of which would, when solved numerically, give substantially the same values. ϕ^n was adopted because of its simplicity if for no other reason. Although this method shows that the porosity function can well be a variable factor, it is believed that the works of Rose and Bruce[13] and others have conclusively shown that the porosity function remains constant while the Kozeny-Carman constant becomes a variable factor in

consolidated media. No attempt was made in the fourth investigation to make use of Fettke's 1938 data because of previous inconsequential results.

In all of the author's four investigations, it has been shown that the procedures, equations, and constants applicable to the empirical formulations derived from the studies of unconsolidated porous media do not hold true in the case of consolidated porous media. However, it also has been shown that for a given or particular consolidated porous rock formation, the empirical formulations apply when the normal constants are changed to those values which fit the situation, as in the case of this study of the consolidated Bradford Third sand. The evidence does not conclusively prove whether the porosity function should be a variable or a constant factor in permeability, porosity and grain size relationship formulations. Much more investigation and research must be carried out along these lines before a conclusion can be reached as to which theory is correct, although as mentioned in the preceding paragraph present evidence points to the porosity function being a constant factor. It is very possible that both the porosity function and the Kozeny-Carman constant might be found to be variables in the case of consolidated porous media.

V. CONCLUSIONS

The field of small particle statistics and measurements is extensive, complex and relatively unknown. For these reasons, the study of particle size in relation to permeability and porosity of porous media is, of itself, extensive, complex and relatively unknown, especially in the case of consolidated porous media. The author agrees with Muskat[21] who stated, in effect, that the use of small particle statistics and measurements in determinations of permeability was so complex and unresolved that the presently utilized methods of determining permeability and porosity far outweigh any method using grain size or its attendant properties to determine same. However, the author feels that it is essentially important to intensively study the field of particle size in order to gain a thorough knowledge and understanding of macroscopic fluid flow in porous media in the light of grain size and its attendant factors of packing, uniformity of size and shape, and degree of lithification, all of which affect fluid flow in such media.

Through an intensive study of this nature, it is possible that some day a formulation will be developed which will truly hold under all conditions and in any circumstances whereby the physical properties of permeability, porosity, and grain size with its attendant factors, can be successfully correlated. The four methods of correlation employed in this study can be used in a restricted degree when applied to the consolidated porous media of particular reservoirs or of sections of the reservoirs only after considerable research to determine the proper constants applicable in each case. In the same vein, and of considerable interest, is the method developed by Ryder[22] for making permeability

measurements without the use of core samples by means of an empirically developed formula, good only for one particular sand, which uses only determined weights of sieved sand particles which have been passed through a series of U. S. Standard sieves.

Another possible investigation or method, suggested to, but untried by the author, would be to plot the results of sieving data from a core sample (per cent by weight of sample through and on each screen size) versus the average screen size for that per cent by weight. From the resultant curve, select a "representative" per cent value and determine the corresponding average screen size or presumed average grain diameter. Using equation (2) of Section III of this study, determine which per cent by weight value and corresponding screen size or presumed average grain size can be used to satisfy the equation, and then determine whether or not this same per cent value can be used in connection with all core sample data from cores taken from the same formation or section of the formation.

A summation of the results of this study does not reflect the original purpose of the study. It was thought that an evaluation could be made of the interrelationships of the physical properties of porous rock, permeability, porosity and grain size, as well as a modification of existing formulations used to express these interrelationships of physical properties. By using data from highly consolidated porous media, it was hoped that if a modification were developed for effectively expressing the interrelationship of permeability, porosity, and grain size, it would hold under any circumstances and conditions for any type medium, either consolidated or unconsolidated. However, the complexities of grain size study, the dearth of sufficient and accurate data, and the apparent nonconformity of application of modified formulations to consolidated porous

media data made such endeavors impractical and impossible.

It is believed that the results of this study, although meager, can contribute some benefits in a better understanding of the problem. The ratio of pore volume to grain size diameter has been shown to decrease markedly with an increase in cementation and a corresponding decrease in grain size such as found in a consolidated porous media like the Bradford Third sand. The value of the Kozeny-Carman constant has been shown to increase to very high values in consolidated porous media. This is probably due to the large increase in values of the tortuosity factor which becomes greater as the rock becomes more consolidated because of an increase in the effective length of flow path. Probably the most important contribution of this study is the realization of the importance of the Kozeny-Carman equation and its modifications when applied to problems of fluid flow through porous media. It holds great possibilities, when subject to the proper modification, in its application to studies of this type and gives promise of eventual solution of the many problems of fluid flow still facing the petroleum industry.

APPENDIX I

TABLE I

Fettke's 1934 Data (K is Experimental)

(1) Grain Size d (cm)	(2) Permeability K (Darcies)	(3) Porosity Ø (per cent)	(4) K/Ø	(5) $(K/Ø)^{\frac{1}{2}}$	(6) Constant x 10^{-4}	(7) d_c = $C(K/Ø)^{\frac{1}{2}}$ (cm) x 10^{-5}
.0092	.0005	10.8	.00460	.0678	5.619	3.81
.0092	.0005	12.5	.00400	.0633	"	3.56
.0093	.0034	14.5	.02340	.1530	"	8.60
.0094	.0082	15.8	.05190	.2278	"	12.80
.0099	.0012	13.4	.00895	.0946	"	5.31
.0106	.0039	16.8	.02320	.1523	"	8.57
.0109	.0037	15.2	.02440	.1562	"	8.78
.0110	.0039	16.2	.02410	.1552	"	8.74
.0110	.0139	20.2	.06890	.2625	"	14.75
.0121	.0213	21.6	.09870	.3142	"	17.60
.0128	.0366	22.7	.16100	.4012	"	22.50
.0132	.0323	21.7	.14900	.3860	"	21.60
.0135	.0226	14.7	.01770	.1330	"	7.47
.0139	.0017	15.0	.01130	.1063	"	5.97

Fettke's 1934 Data (K is Calculated)

.0092	.0004	10.8	.00370	.0608	5.619	3.42
.0092	.0010	12.5	.00800	.0894	"	5.02
.0093	.0024	14.5	.01660	.1288	"	7.23
.0094	.0038	15.8	.02400	.1549	"	8.74
.0099	.0015	13.4	.01200	.1095	"	6.15
.0106	.0045	16.8	.02680	.1637	"	9.20
.0109	.0031	15.2	.02040	.1428	"	8.02
.0110	.0044	16.2	.02720	.1649	"	9.26
.0110	.0154	20.2	.07630	.2762	"	15.50
.0121	.0225	21.6	.10400	.3225	"	18.10
.0128	.0298	22.7	.13130	.3624	"	20.40
.0132	.0231	21.7	.10600	.3256	"	18.30
.0135	.0026	14.7	.01770	.1330	"	7.46
.0139	.0029	15.0	.01930	.1389	"	7.80

Fettke's 1938 Data (K is Experimental)

.0041	.00278	13.3	.02090	.1446	5.619	8.12
.0042	.00047	12.1	.00389	.0624	"	3.50
.0042	.00181	12.8	.01410	.1187	"	6.68
.0042	.00550	14.3	.03840	.1960	"	11.00
.0042	.00912	15.2	.05980	.2445	"	13.72
.0044	.00534	14.3	.03740	.1934	"	10.86
.0047	.02751	15.7	.17530	.4187	"	23.50
.0051	.02770	15.7	.17630	.4199	"	23.60

TABLE II

Fettke's 1934 Data (K is Experimental)

(1) Grain Size d (cm)	(2) Permeability K (Darcies)	(3) Porosity ϕ (per cent)	(4) $1 - \phi$	(5) $(\phi/K)^{1/2}$	(6) Specific Surface, S_o (cm^{-1})	(7) 1/d
.0092	.0005	10.8	.892	14.70	8000	108.6
.0092	.0005	12.5	.875	15.81	10,170	108.6
.0093	.0034	14.5	.855	6.53	4980	107.4
.0094	.0082	15.8	.842	4.39	3700	106.3
.0099	.0012	13.4	.866	10.56	7350	101.0
.0106	.0039	16.8	.832	6.57	5970	94.4
.0109	.0037	15.2	.848	6.41	5160	91.8
.0110	.0039	16.2	.838	6.45	5610	91.0
.0110	.0139	20.2	.798	3.81	4340	91.0
.0121	.0213	21.6	.784	3.18	3950	82.7
.0128	.0366	22.7	.773	2.49	3290	78.1
.0132	.0323	21.7	.783	2.59	3230	75.8
.0135	.0026	14.7	.853	7.52	5825	74.1
.0139	.0017	15.0	.850	9.39	7460	71.9

Fettke's 1934 Data (K is Calculated)

.0092	.0004	10.8	.892	16.43	8960	108.6
.0092	.0010	12.5	.875	11.18	7175	108.6
.0093	.0024	14.5	.855	7.77	5930	107.4
.0094	.0038	15.8	.842	6.45	5450	106.3
.0099	.0015	13.4	.866	9.46	6580	101.0
.0106	.0045	16.8	.832	6.12	5630	94.4
.0109	.0031	15.2	.848	7.00	5650	91.8
.0110	.0044	16.2	.838	6.07	5275	91.0
.0110	.0154	20.2	.798	3.62	4120	91.0
.0121	.0225	21.6	.784	3.10	3840	82.7
.0128	.0298	22.7	.773	2.76	3645	78.1
.0132	.0231	21.7	.783	3.07	3830	75.8
.0135	.0026	14.7	.853	7.52	5825	74.1
.0139	.0029	15.0	.850	7.19	5700	71.9

Fettke's 1938 Data (K is Experimental)

.0041	.00278	13.3	.867	6.91	4770	244.0
.0042	.00047	12.1	.879	16.06	9960	238.0
.0042	.00181	12.8	.872	8.41	5560	238.0
.0042	.00550	14.3	.857	5.10	3840	238.0
.0042	.00912	15.2	.848	4.08	3300	238.0
.0044	.00534	14.3	.857	5.18	3390	227.0
.0047	.02751	15.7	.843	2.39	2000	212.5
.0051	.02770	15.7	.843	2.33	1995	196.0

TABLE III

Fettke's 1934 data (K is experimental)

(1) Grain Size d (cm)	(2) Permeability K (Darcies)	(3) Porosity ϕ (per cent)	(4) ϕ^3	(5) $(1 - \phi)^2$	(6) K/d^2	(7) $\phi^3/(1-\phi)^2$ X 1.013 X 10^8
.0092	.0005	10.8	.00138	.795	5.91	176000
.0092	.0005	12.5	.00195	.765	5.91	259000
.0093	.0034	14.5	.00305	.731	39.30	423000
.0094	.0082	15.8	.00394	.708	93.20	564000
.0099	.0012	13.4	.00241	.750	12.25	326000
.0106	.0039	16.8	.00474	.692	34.80	694000
.0109	.0037	15.2	.00350	.720	31.20	493000
.0110	.0039	16.2	.00425	.704	32.20	613000
.0110	.0139	20.2	.00824	.634	115.00	1320000
.0121	.0213	21.6	.01007	.615	146.00	1660000
.0128	.0366	22.7	.01180	.597	223.00	2000000
.0132	.0323	21.7	.01023	.614	185.60	1695000
.0135	.0026	14.7	.00318	.727	14.30	443000
.0139	.0017	15.0	.00338	.722	8.80	474000

Fettke's 1934 data (K is calculated)

.0092	.0004	10.8	.00138	.795	4.72	176000
.0092	.0010	12.5	.00195	.765	11.80	259000
.0093	.0024	14.5	.00305	.731	27.80	423000
.0094	.0038	15.8	.00394	.708	43.00	564000
.0099	.0015	13.4	.00241	.750	15.30	326000
.0106	.0045	16.8	.00474	.692	40.20	694000
.0109	.0031	15.2	.00350	.720	26.10	493000
.0110	.0044	16.2	.00425	.704	36.40	613000
.0110	.0154	20.2	.00824	.634	128.20	1320000
.0121	.0225	21.6	.01007	.615	154.00	1660000
.0128	.0298	22.7	.01180	.597	182.00	2000000
.0132	.0231	21.7	.01023	.614	132.80	1695000
.0135	.0026	14.7	.00318	.727	14.30	443000
.0139	.0029	15.0	.00338	.722	15.00	474000

Fettke's 1938 data (K is experimental)

.0041	.00278	13.3	.00235	.752	165.50	317000
.0042	.00047	12.1	.00177	.774	25.40	232000
.0042	.00181	12.8	.00210	.760	103.00	280000
.0042	.00550	14.3	.00292	.735	313.00	403000
.0042	.00912	15.2	.00351	.720	518.00	494000
.0044	.00534	14.3	.00292	.735	275.00	403000
.0047	.02751	15.7	.00387	.710	1244.00	552000
.0051	.02770	15.7	.00387	.710	1065.00	552000

TABLE IV

(K is experimental)

(1) Permeability K (darcies)	(2) Grain Size d (cm)	(3) K/d^2	(4) K/d^2 / 1.013×10^8	(5) Porosity ϕ (per cent)
.0005	.0092	5.91	5.83×10^{-8}	10.8
.0005	.0092	5.91	5.83	12.5
.0034	.0093	39.30	38.70	14.5
.0082	.0094	92.80	91.60	15.8
.0012	.0099	12.25	12.08	13.4
.0039	.0106	34.80	34.30	16.8
.0037	.0109	31.20	30.75	15.2
.0039	.0110	32.20	31.75	16.2
.0139	.0110	115.00	113.50	20.2
.0213	.0121	146.00	144.00	21.6
.0366	.0128	223.00	220.00	22.7
.0323	.0132	185.60	183.00	21.7
.0026	.0135	14.30	14.10	14.7
.0017	.0139	8.80	8.68	15.0

(K is calculated)

.0004	.0092	4.72	4.66×10^{-8}	10.8
.0010	.0092	11.80	11.64	12.5
.0024	.0093	27.80	27.40	14.5
.0038	.0094	43.00	42.40	15.8
.0015	.0099	15.30	15.10	13.4
.0045	.0106	40.20	39.60	16.8
.0031	.0109	26.10	25.70	15.2
.0044	.0110	36.40	35.90	16.2
.0154	.0110	128.20	126.60	20.2
.0225	.0121	154.00	152.00	21.6
.0298	.0128	182.00	179.50	22.7
.0231	.0132	132.80	131.00	21.7
.0026	.0135	14.30	14.10	14.7
.0029	.0139	15.00	14.80	15.0

A P P E N D I X II

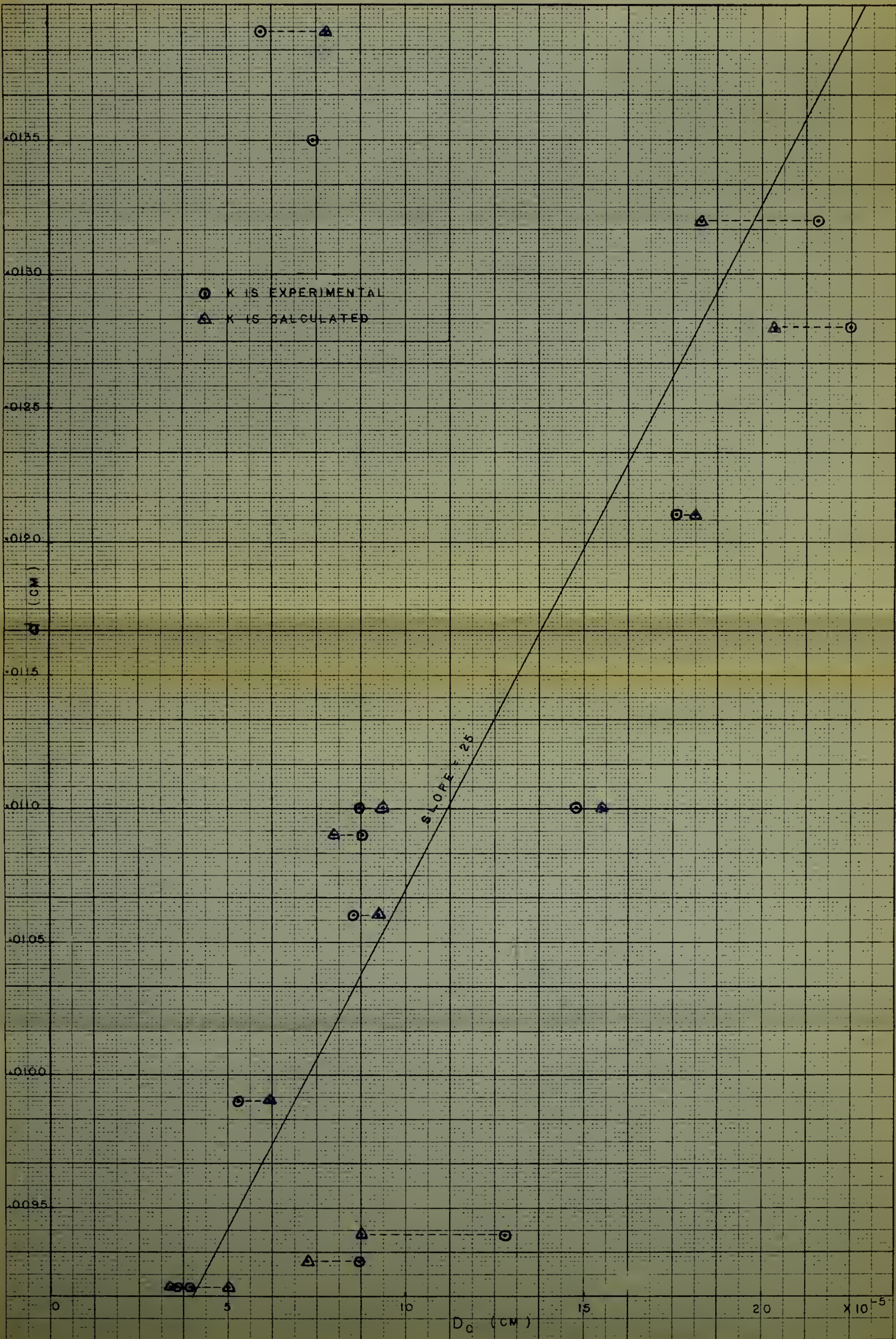
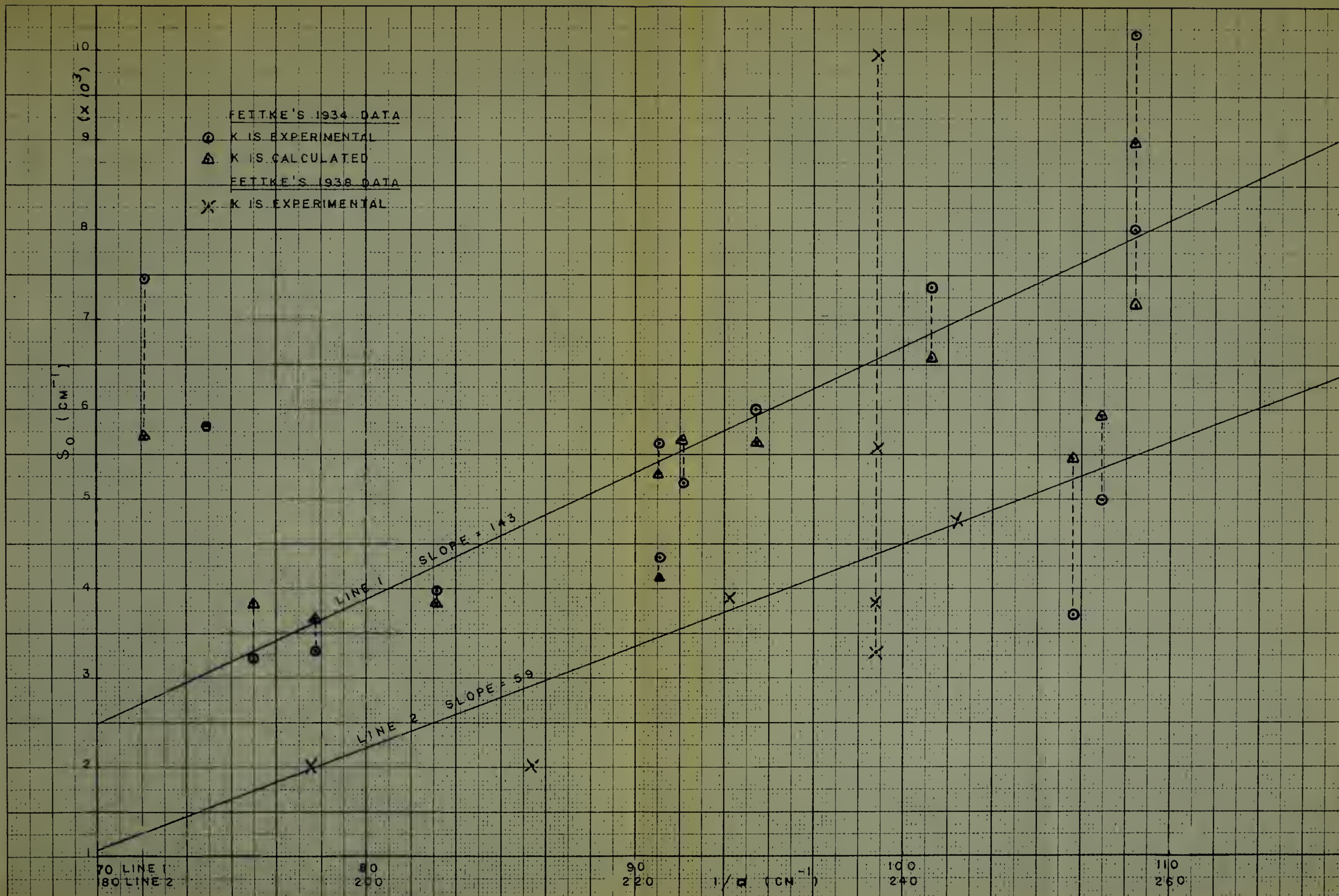

.0135
.0130
.0125
.0120
.0115
.0110
.0105
.0100
.0095
d (CM)
K IS EXPERIMENTAL
K IS CALCULATED
SLOPE = 25
0
5
10
15
20
X 10^-5
D_c (CM)

FIGURE I

FIGURE 2



FETTKE'S 1934 DATA
K IS EXPERIMENTAL
K IS CALCULATED
FETTKE'S 1938 DATA
K IS EXPERIMENTAL
LINE 1 SLOPE = 143
LINE 2 SLOPE = 59
S_0 (CM^{-1}) $(\times 10^3)$
$1/d$ (CM^{-1})
70 LINE 1
180 LINE 2
80
200
90
220
100
240
110
260
1
2
3
4
5
6
7
8
9
10

FIGURE 3

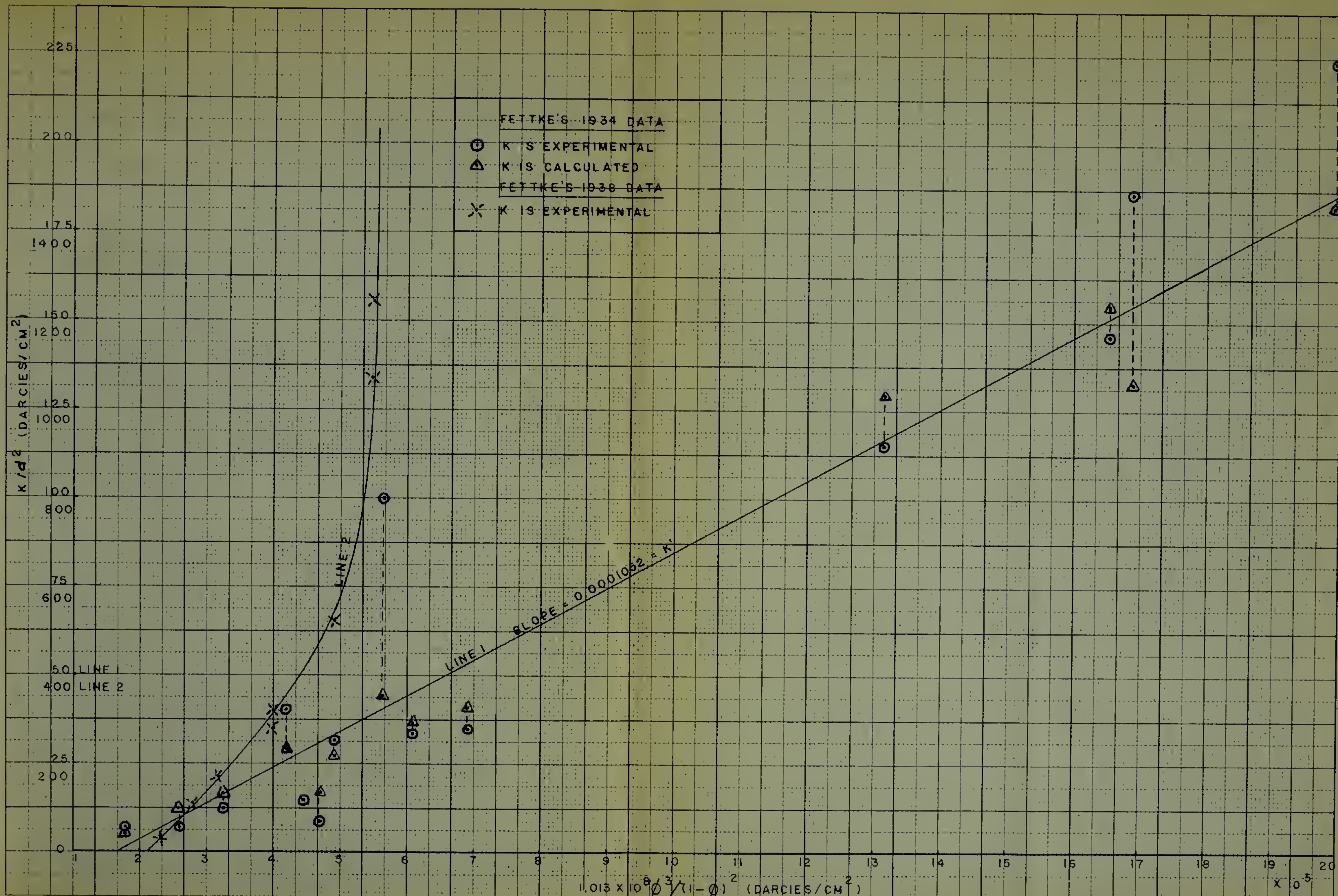
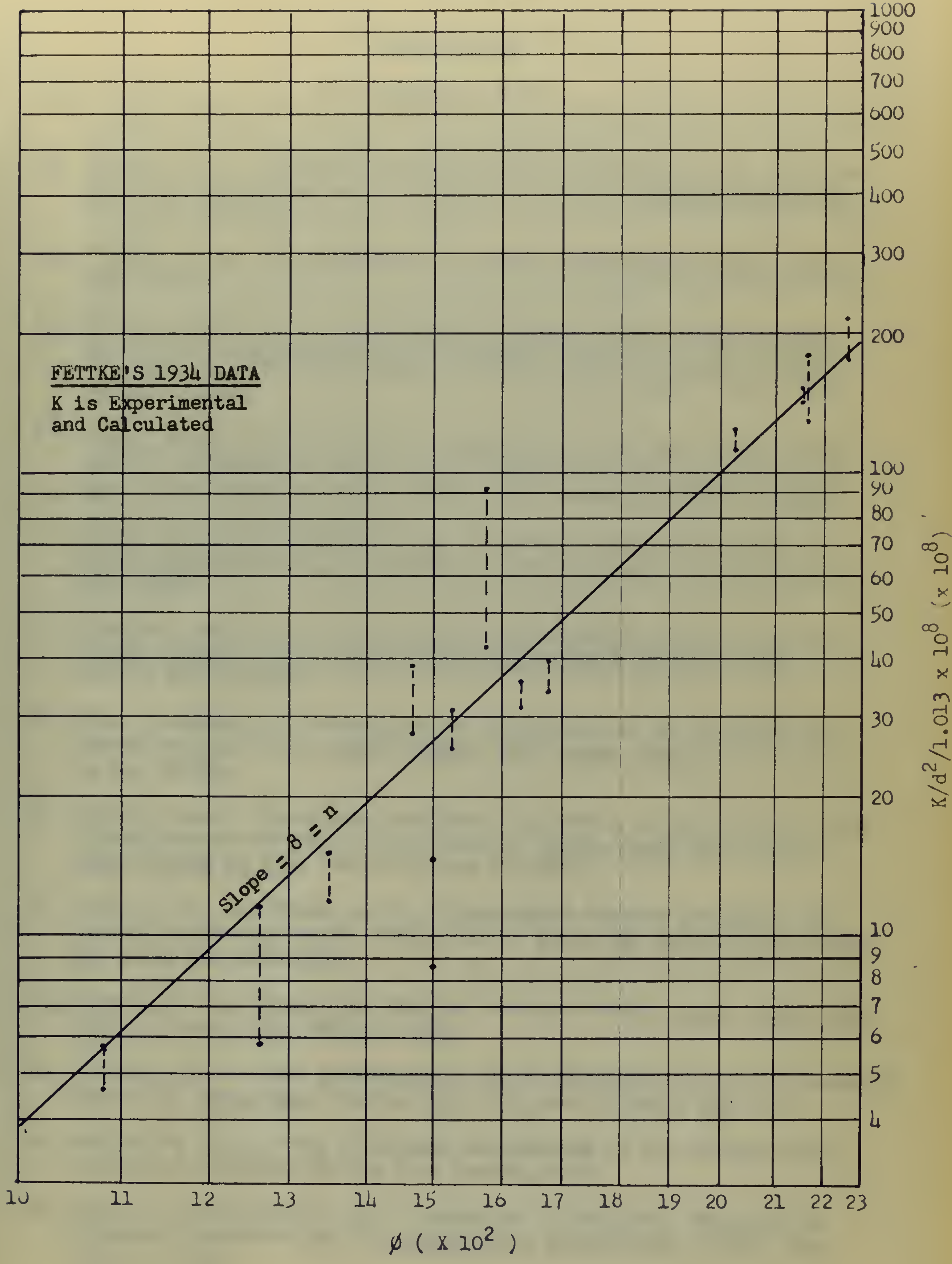

FETTKE'S 1934 DATA
K IS EXPERIMENTAL
K IS CALCULATED
FETTKE'S 1938 DATA
K IS EXPERIMENTAL
LINE 2
LINE 1
SLOPE = 0.0001052 = K'
LINE 1
LINE 2
K/d2 (DARCIES/CM2)
1.013 X 10^8 ϕ^3/(1-ϕ)^2 (DARCIES/CM^2)
X 10^-5
225
200
175
1400
150
1200
125
1000
100
800
75
600
50
400
25
200
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

FIGURE 4



FETTKE'S 1934 DATA
K is Experimental
and Calculated
Slope = 8 = n
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
10
11
12
13
14
15
16
17
18
19
20
21
22
23
$K/d^2/1.013 \times 10^8$ ($\times 10^8$)
ϕ (X 10^2)

BIBLIOGRAPHY

A - References Cited

1. Fettke, C. R., "Physical Characteristics of Bradford Sand, Bradford Field, Pa., and Relation to Production of Oil," Bulletin AAPG, 18, 1934, pp. 191-211.

2. Fettke, C. R., "The Bradford Oil Field," Pennsylvania Geol. Survey, M-21, 1938.

3. D'Arcy, Henri P. G., "Les Fontains Publiques de la Ville de Dijon, Exposition et Application des Principles a Suivre et des Formules a Employer dans les Questions de Distribution d'Eau," 1856, Paris: Victor Dalmont.

4. Hazen, Allen, "Some Physical Properties of Sands and Gravels with Special Reference to their Use in Filtration," 24th Annual Report, Mass. State Board of Health, 1892, Pub. Document 34, pp. 539-556.

5. Davis, F. T., and Wilsey, E. F., "A Study of the Permeability of Sand," University of Iowa Studies in Engr. Bull. 7, New Series No. 316, 1936.

6. Slichter, Charles S., "Theoretical Investigation of the Motion of Ground Waters," U. S. Geol. Survey, 19th Annual Report, 1899, Part 2, pp. 301-384.

7. King, Franklin H., "Principles and Conditions of the Movements of Ground Waters," U. S. Geol. Survey, 19th Annual Report, 1899, Part 2, pp. 59-294.

8. Kozeny, Josef, "Concerning Capillary Conduction of Water in the Soil (Rise, Seepage and Use in Irrigation)," Sitzb. Akad. Wiss. Wien., Math.-Naturw Kl 136, (Abt IIa), pp. 271-306.

9. Fair, G. M., and Hatch, L. P., "Fundamental Factors Governing the Streamline Flow of Water Through Sand," Jour. Am. Water Works Assoc., 25, 1933, pp. 1551-1565.

10. Carman, P. C., "Fluid Flow Through Granular Beds," Trans. Inst. Chem. Engr., (London) 15, 1937, p. 152.

11. Carman, P. C., "The Determination of the Specific Surface of Powders," Jour. Soc. Chem. Ind. (London) 57, 1938, pp. 225-234; 58, p. 1.

12. Wyllie, M. R. J., "The Historical Development of the Kozeny-Carman Equation," Published by the Pet. Branch, AIME.

13. Rose, W., and Bruce, W. A., "Evaluation of Capillary Character in Petroleum Reservoir Rock,", Journal Pet. Tech.(Trans. AIME), 186, 1949, p. 127.

14. Rose, H. E., "The Measurement of Particle Size in Very Fine Powders," 1953, (London) Constable and Company, Ltd., pp. 78-95.

BIBLIOGRAPHY (Continued)

15. Dallavalle, J. M., "Micromeritics, The Technology of Fine Particles," 1948, New York: Pitman Publishing Corp., p. 47, 275.

16. Wyllie, M. R. J., and Spangler, M. B., "Application of Electrical Resistivity Measurements to Problem of Fluid Flow in Porous Media," Bulletin AAPG, 36, 1952, p. 359.

17. Pirson, S. J., "Elements of Oil Reservoir Engineering," p. 46, New York: McGraw-Hill Book Company, Inc., 1950.

18. Traxler, R. N., and Baum, L. A. H., "Permeability of Compacted Powders - Determination of Average Pore Size," Physics, 7, 1936, pp. 9-14.

19. Nutting, P. G., "The Movements of Fluids in Porous Soils," Journal Franklin Institute, 203, 1927, pp. 314-24.

20. Herdan, G., "Small Particle Statistics," p. 264, 1953, Amsterdam: Elsevier Publishing Company.

21. Muskat, M., "The Flow of Homogeneous Fluids Through Porous Media," 1937, New York: McGraw-Hill Book Co., Inc., p. 74.

22. Ryder, H. M., "Permeability Measurements Without Cores," The Ryder Scott Company, Bradford, Pennsylvania.

B. Related References Not Cited

1. Fancher, G. H., Lewis, J. A., and Barnes, K. B., "Some Physical Characteristics of Oil Sands," *Pennsylvania State College Bulletin, Mineral Ind. Exp. Station, Bulletin 12*, 1933.

2. Rose, W., "Theoretical Generalizations Leading to the Evaluation of Relative Permeability," *Journal Pet. Tech.* (*Trans. AIME*), *186*, 1949, p. 127.

3. Rose, W., and Wyllie, M. R. J., "Specific Surface Areas and Porosities from Photomicrographs," *Bulletin AAPG*, *34*, 1950, p. 1743.

4. Wyllie, M. R. J., and Rose, W. D., "Application of Kozeny-Carman Equation to Consolidated Porous Media," *Nature*, *165*, 1950, p. 972.

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